Re-orientation of Fibres During Mechanical Working

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A simple model is used to calculate the distribution of fibre axes resulting from the deformation of a matrix containing fibres initially randomly oriented. It is shown that for moderately large amounts of deformation, concurrent internal shear is unlikely to make a significant contribution towards the alignment of fibres. After deformation, the density of fibre axes oriented in a particular spatial direction, $dN/d\Omega$ where Ω is a solid angle, is shown to be proportional to the cube of the length of the radius vector from the origin to the surface of the strain ellipsoid.

1. Introduction

Mechanical working has been suggested as a method of aligning fibres, originally oriented randomly, so that their axes lie approximately parallel to the direction in which the working load is to be applied in service, and the fibres exert their maximum strengthening capacity. Extrusion and rolling, amongst other forming processes, have been suggested for this purpose [1-4], and it has been suggested that one or other method of working gives the better fibre alignment [5]. The object of the present work is threefold: (a) to provide a method for the calculation of the distribution of fibre axes in space, resulting from an arbitrary deformation of the matrix containing the fibres, on the basis of simple assumptions; (b) to examine the implications of the resultant distributions on the choice of methods of working intended to produce a desired change in shape and an optimum fibre alignment; (c) to test the validity of these assumptions by use of a model system.

2. Calculation of Fibre Alignment

We assume that: (a) the fibres undergo re-orientation as if they were isolated in the matrix without mutual interference; (b) the fibres are sufficiently short that the deformation of the local matrix in which they are embedded may be considered uniform; (c) the re-orientation of fibres is the same as that of the matrix with which they were initially in contact.

2.1. Fibre Distribution Resulting from Arbitrary Deformation: Zero Volume Change

In three dimensions, the density of fibre axes oriented in a particular direction may be represented by $dN/d\Omega$ where Ω is a solid angle. We shall assume an initially random distribution of axes in three dimensions,

$$\frac{dN}{d\Omega} = \frac{1}{2\pi},$$

so that

$$\int \left(\frac{dN}{d\Omega}\right) d\Omega = 1,$$

since all possible fibre orientations may be considered as originating at the origin of a unit sphere, and passing through a hemispherical surface. Hence the integral is performed over a solid angle of 2π.

Any fibre whose axis threads through part of the surface of the sphere before deformation will thread through the corresponding portion of the distorted surface after deformation. The problem is thus to determine the solid angle $d\Omega'$ subtended by an area element dS' (corresponding to $dS = d\Omega$ in the undeformed sphere) after deformation.

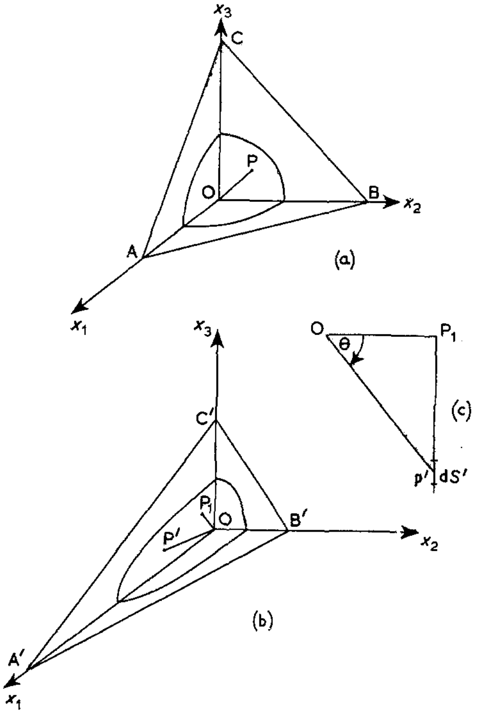


Figure 1 (a) Unit sphere in an undeformed matrix, showing tangent plane at P. (b) Strain ellipsoid, showing tangent plane at P′, corresponding to the point P in the undeformed body. (c) Diagram to show the solid angle subtended by the area element dS' corresponding to dS in the undeformed body.

In fig. 1 the point P on the unit sphere has coordinates (x_1, x_2, x_3) such that $x_1^2 + x_2^2 + x_3^2 = 1$, referred to the principal axes of strain*, and a fraction dN of fibres thread through an area dS in the surface around P, such that

$$\frac{dN}{dS} = \frac{1}{2\pi}\,.$$

After deformation, the coordinates of P, now the point P′, become (n_1x_1, n_2x_2, n_3x_3) where n_1, n_2 and n_3 are the extension ratios ($n_1n_2n_3 = 1$, since there is no volume change during deformation).

OP_1 is the normal to A′ B′ C′. Since the volume of the tetrahedron ABCO equals the volume of A′ B′ C′ O′,

$$\frac{dS'}{dS} = \frac{\text{area A' B' C'}}{\text{area ABC}} = \frac{OP}{OP_1}\,.$$

That is, $$\frac{dS'}{dS} = \frac{1}{OP_1}\,.$$

Now in fig. 1c, the solid angle $d\Omega'$, subtended by dS' at O, is $\dfrac{dS'\cos\theta}{(OP')^2}$, where $\cos\theta = \dfrac{OP_1}{OP'}$.

Hence $$\frac{\cos\theta}{(OP')^2} = \frac{OP_1}{(OP')^3}\,.$$

Therefore $$d\Omega' = \left(\frac{dS}{OP_1}\right)\left(\frac{OP_1}{(OP')^3}\right) = dS/(OP')^3\,.$$

Hence $$\frac{dN}{d\Omega'} = \frac{dN}{dS}(OP')^3$$

or $$\left(\frac{dN}{d\Omega}\right)' = \frac{r^3}{2\pi} \qquad (1)$$

after deformation, where $r = OP'$.

Hence the density of fibres oriented parallel to a given direction in the deformed body is proportional to the cube of the length of the radius vector from the origin to the strain ellipsoid's surface. This is true for fibres originally distributed randomly in three dimensions.

Similarly it may be shown that, in the case of fibres oriented randomly in two dimensions, the density of fibres oriented with axes in a particular direction is proportional to the square of the distance from the origin to the strain ellipse, provided the strain is confined to the plane of the fibre distribution.

2.1.1. Plane Strain

If in plane strain we define the strain at a particular point by the extension ratio n and the concurrent internal shear γ (fig. 2), then the equation to the strain ellipse is:

$$\frac{x^2}{n^2} - \frac{2\gamma}{n^2}xy + \left(\frac{\gamma^2}{n^2} + n^2\right)y^2 = 1$$

and we obtain:

$$r = \left[\left(n^2 + \frac{\gamma^2}{n^2} - \frac{1}{n^2}\right)\sin^2\theta + \frac{1}{n^2} - \frac{\gamma}{n^2}\sin 2\theta\right]^{-\frac{1}{2}}.$$

The resulting distribution of fibre axes is shown in fig. 3 for $n = 1$, 2 and 4: a factor of $1/\pi$ has been introduced to make

*None of the principal axes of strain necessarily coincides with the axis of, for example, a drawn wire or extruded rod. The principal axes of strain may also rotate during deformation, but this may be neglected if it is assumed, as here, that the original distribution is isotropic.

$$\int_{-\pi/2}^{+\pi/2} \left(\frac{dN}{d\theta}\right) d\theta = 1$$

since all fibre axes may be considered to lie between $-\pi/2$ and $+\pi/2$.

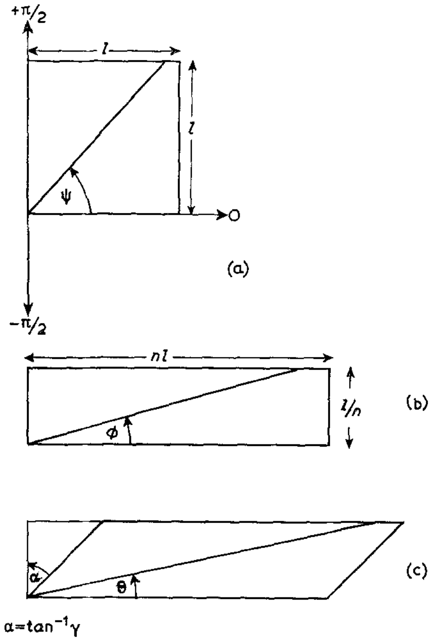


Figure 2 Rotation of a fibre during plane strain. $\theta = 0$ represents perfect fibre alignment: plane strain is considered as taking place with an extension ratio n and a concurrent internal shear γ. From the figure, $\phi = \cot^{-1}(n^2 \cot \psi)$ and $\theta = \cot^{-1}(n^2 \cot \psi + \gamma)$.

The following points are apparent from these curves in fig. 3: (a) for simple shear (fig. 3a) the maximum density of fibre axes does not coincide with the specimen axis, but rotates towards it as the shear increases: at the same time, the maximum value of $dN/d\theta$ rises; (b) for small values of n (fig. 3b) γ is effective in increasing the maximum value of $dN/d\theta$: the position of the peak moves away from $\theta = 0$ as γ increases, but then moves back once more at large γ; (c) at large n (fig. 3c) γ has little effect either on the position or magnitude of the peak value of $dN/d\theta$.

2.1.2. Axisymmetric Strain

For axisymmetric deformation (for example, the extrusion of rod), the distribution is required with respect to θ, where θ is the inclination of the fibre to the extrusion axis (fig. 4)*. In fig. 4, the sides of the element at N subtend angles of $d\theta$ and $d\phi$ at the origin: the area element subtends a solid angle $d(d\Omega) = \sin\theta \, d\theta \, d\phi$. The fraction of fibres having axes lying between θ and $\theta + d\theta$, over all values of ϕ, is therefore, from equation 1:

$$dN = \frac{1}{2\pi} \int_{\phi=0}^{\phi=2\pi} r^3 \sin\theta \, d\theta \, d\phi \, ,$$

where r is the distance from the origin to the surface of the strain ellipsoid, in the same spatial direction as ON.

If the extrusion ratio is n, then the equation to the strain ellipsoid, without taking account of concurrent internal shear, is

$$(x^2/n^2) + ny^2 + nz^2 = 1$$

since the strains in the radial (y-direction) and circumferential (z-direction) are equal. The concurrent internal shear, which is equivalent to telescopic deformation of the extruded rod, is parallel to the xz-plane in the x-direction: the equation to the strain ellipsoid becomes:

$$\frac{x^2}{n^2} - \frac{2\gamma}{n^2} xy + \left(\frac{\gamma^2}{n^2} + n\right) y^2 + nz^2 = 1 \, .$$

Transformation to the spherical polar co-ordinates of fig. 4 gives:

$$r = \left[\frac{1}{n^2}\cos^2\theta - \frac{\gamma}{n^2}\sin 2\theta \cos\phi + \left(\frac{\gamma^2}{n^2} + n\right)\sin^2\theta\cos^2\phi + n\sin^2\theta\sin^2\phi\right]^{-\frac{1}{2}} .$$

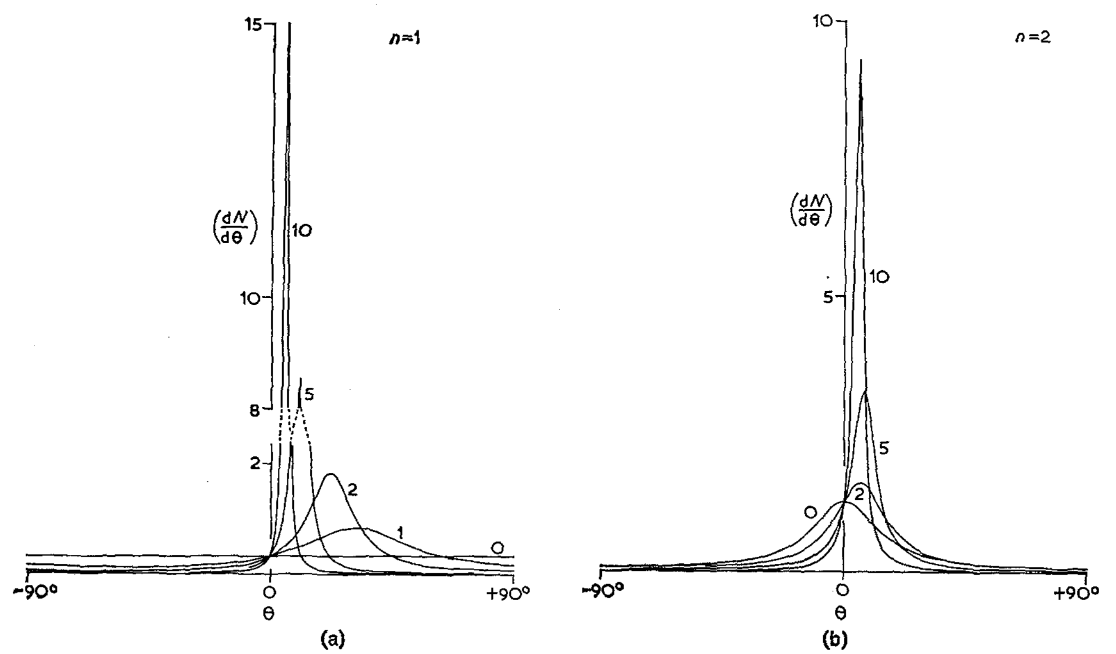
The resultant distribution has been computed for a number of values of n and γ and is shown in fig. 5 for $n = 5$. Again it can be seen that concurrent internal shear contributes little to the fibre alignment.

2.3. Finite Volume Change

Attempts have been made to produce fibre-reinforced materials by incorporation of fibres in powder [1, 2]. The fibres and powder are compacted and then subjected to some working process such as extrusion, to align the fibres with the axis of the extruded rod.

If we assume that the fibres are randomly disposed in the uncompacted powder, the compaction, if it occurs in the same direction as

*For an initially three-dimensional fibre distribution, the distinction between negative and positive θ, which was made above in the case of the plane strain deformation of a two-dimensional distribution, is no longer possible. Hence the fibre distributions range from $\theta = 0$ to $\theta = \frac{1}{2}\pi$.



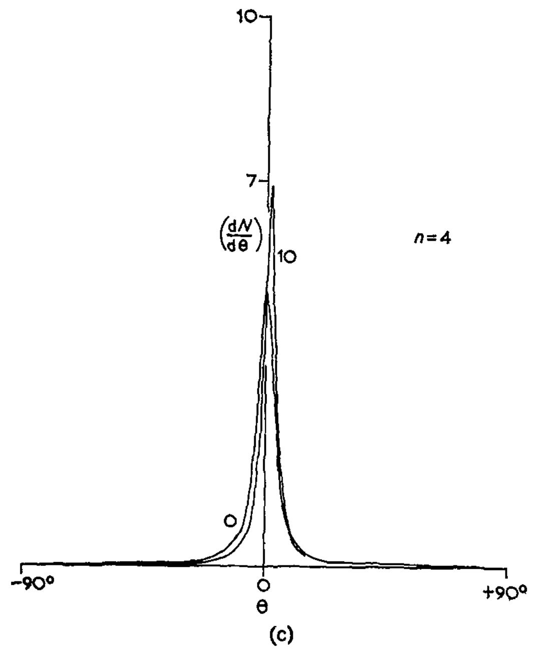


Figure 3 Frequency distribution curves for fibre orientations resulting from the plane strain deformation of a matrix containing an original random two-dimensional fibre distribution in the plane of strain. Extension ratio, n: (a) 1; (b) 2; (c) 4. The corresponding value of γ at each extension is shown beside each curve.

the subsequent extrusion, rotates the fibres away from the desired orientation. The distribution of fibre axes is therefore not random in the extrusion billet. The effect of such volume changes may be taken into account by use of a hypothetical process to yield the same final external shape and concurrent internal shearing.

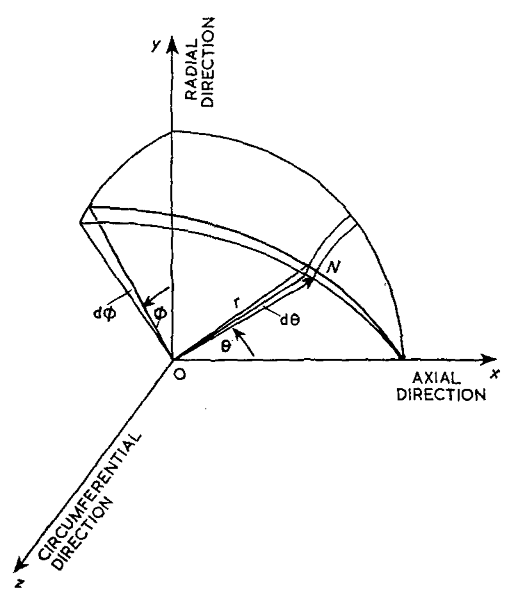


Figure 4 Cartesian co-ordinates and spherical polar co-ordinates in axisymmetric deformation. The x-direction corresponds to the specimen axis, the z- and y-axes to the circumferential and radial directions respectively. Concurrent internal shear is parallel to the xz-plane in the x-direction.

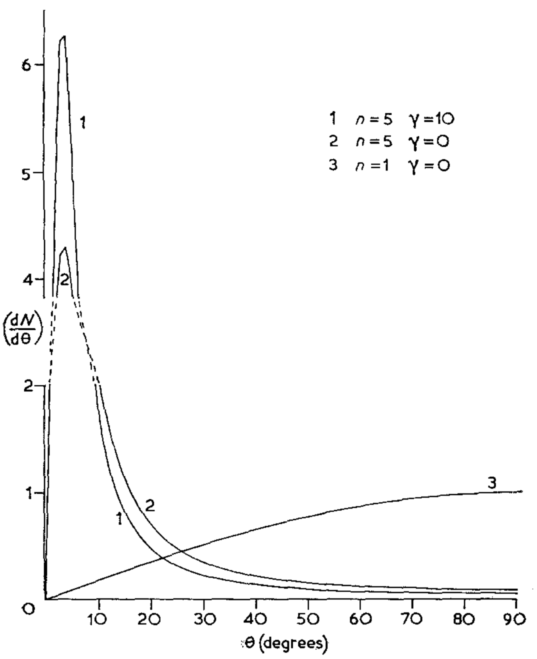


Figure 5 Frequency distribution curves for fibre orientation resulting from axisymmetric deformation. Extension ratio $n = 5$; redundant internal shear, $\gamma = 0$, 10. θ is the angle between the specimen axis and the fibre axis. The initial isotropic distribution is also shown.

We imagine that the volume change occurs first without change in shape, and that extrusion then occurs at an extrusion ratio such that the final extruded rod diameter is obtained. Re-orientation of the fibres then occurs only during the hypothetical extrusion process (accompanied by the effects of concurrent internal shear).

Let f be the ratio of the compacted to the uncompacted volume, n_f the actual extrusion ratio and γ the concurrent shearing, as defined in the previous section. Then the distribution of fibre axes becomes the same as that resulting from the extrusion of a solid with random fibre orientation, with an effective extrusion ratio $n = n_f f^{2/3}$, accompanied by concurrent internal shear γ. Thus if $f = 0.5$, $n \simeq 0.6\ n_f$, then the extrusion ratio must be increased by a factor of ~ 1.7 over that which would yield a given fibre distribution in solid material.

3. Experimental

In order to ascertain whether the assumptions made in section 2 are valid, the rotation of fibres during mechanical working has been studied experimentally for the plane strain extrusion of Plasticine containing short lengths of 30 swg copper wire. The billet was prepared as a

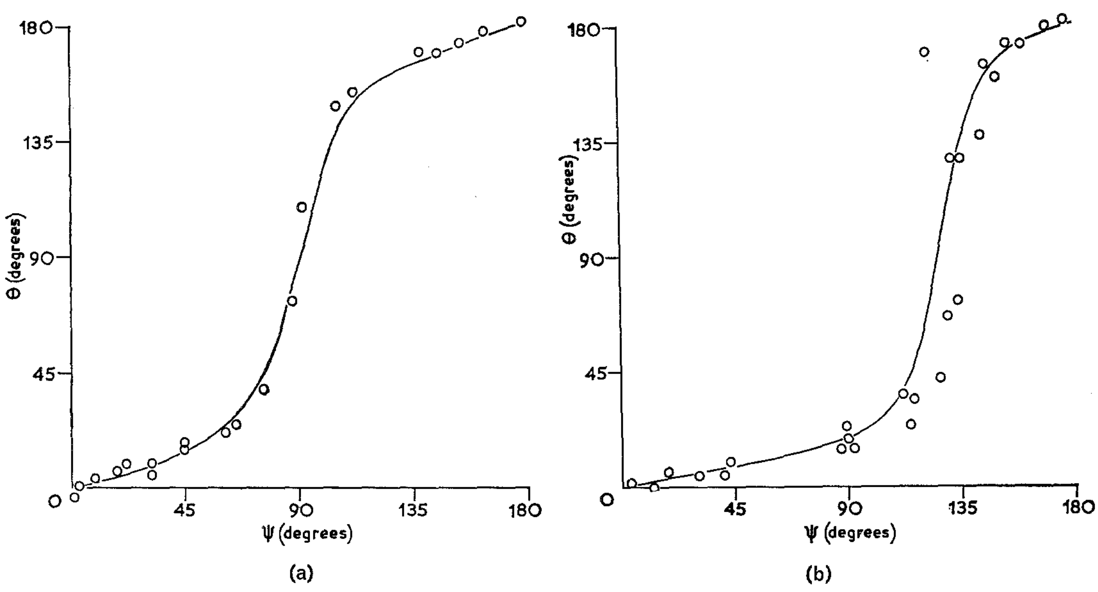


Figure 6 Relationship between initial and final orientation of short copper wires embedded in Plasticine. The solid curves correspond to the relationship calculated from the observed extension and concurrent internal shear, namely: (a) $n = 2$, $\gamma = 0$; (b) $n = 2$, $\gamma = 2.9$.

sandwich, approximately $\frac{1}{2}$ in. thick $\times$ 3in. wide $\times$ 8 in. long, containing the copper wires, approximately $\frac{1}{8}$ in. long, in the mid-plane. The wires were distributed in three lines parallel to the length of the billet (the extrusion direction): one along the axis of extrusion and the other two lines at a distance of $\frac{1}{2}$ in. from the sides. The orientations of the wires were found by radiography of the billet before and after extrusion: these are shown in fig. 6 for 2:1 plane strain extrusion through a square die. Lubrication was effected by use of a wallpaper adhesive. The wires along the axis of extrusion were embedded in a matrix for which $n = 2$, $\gamma = 0$: those near the billet edge were embedded in a matrix for which $n = 2$, $\gamma = 2.9$. The value of γ was determined from the rotation of wires originally at 90° to the axis of extrusion. For plane strain, the relation between ψ, the angle at which the fibre was originally inclined to the specimen axis, and θ, the angle after deformation (fig. 2), is $\theta = \cot^{-1}(n^2 \cot \psi + \gamma)$.

The curves in fig. 6 were calculated for the appropriate values of n and γ: it can be seen that there is agreement between the observed behaviour of the wires and that expected from the simple theory.

4. Discussion

The experimental evidence supports the view that the re-orientation of short fibres may be described with the simple assumptions made in section 2. These assumptions cease to be valid if the fibres are long in comparison with the dimensions over which the deformation is approximately uniform or if the volume fraction of fibres is sufficiently large to cause mutual interference to their movement. The analysis requires a knowledge of the strains produced in the worked material; the final orientation of any given fibre depends only on the final strain. Thus, the path by which the final strains are reached does not influence the final distribution of fibre orientations, although it will, of course, affect the way in which the final distribution is attained.

The distribution curves derived form a basis for assessing the degree of working required to produce a desired degree of fibre alignment. Concurrent internal shearing becomes of decreasing importance in causing alignment in axisymmetric and in plane strain forming processes as the reduction in area increases. In addition, from the evidence of deformed grids, appreciable internal shearing is found only near the outside surface of an extruded or drawn rod, becoming zero along the axis. The effect of this sub-surface shear should be apparent in improved fibre alignment at low extension ratios which in any case are not effective in aligning fibres throughout the bulk of the material. This has been reported in powder compacts extruded at an estimated effective extrusion ratio of 2.5-4 [4]. Provided the desired shape change is achieved, there seems to be little reason for choosing any particular method of working on the ground of superior performance in fibre alignment, unless very large concurrent shearing is involved.

From this it follows that tool geometry (for example, the die angle in extrusion) will have little effect on fibre alignment, since it affects only the concurrent internal shear.

Recently the rotation of fibres contained in viscous liquid has been treated hydrodynamically [6]. The rotation of the fibre is caused by a velocity gradient in the liquid (it is, of course, the existence of a velocity gradient which causes the strain), and the rate of rotation is considered to be controlled by the moment of inertia of the fibre. The rotation is then dependent on the aspect of the fibre. However, such inertial effects are unlikely to be important at the low Reynolds numbers obtaining in the methods so far used to align fibres, and a more satisfactory treatment would be in terms of the strain ellipsoid.

5. Conclusions

On the basis of simple assumptions, it has been shown that after deformation the density of short fibres oriented in a particular direction is proportional to r^3 (where r is the length of the radius vector from the origin to the surface of the strain ellipsoid) for an initially random three-dimensional distribution of fibres.

Curves for the distribution of fibre axis directions after plane strain and after axisymmetric deformation have been presented: concurrent internal shear becomes of decreasing importance in contributing to fibre alignment as the degree of working increases.

Acknowledgement

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